A STATISTICAL INTERPRETATION OF THE J. C. MARTIN RELATIONSHIP FOR BREAKDOWN OF INSULATORS IN VACUUM*

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Abstract

We discuss the application of Weibull statistics and the J. C. Martin empirical relationship to estimate the cumulative probability of failure of an electrical insulator stressed in vacuum with submicrosecond electrical pulses. We display experimental data and analyze it with Weibull statistics. We apply our results to the design of a much larger insulator (~ 10^4 cm^2) with a desired (10%) cumulative probability of failure.

Introduction

Insulators separating the liquid dielectric from the vacuum in a pulsed power particle beam, high density plasma, or x-ray generator are the weakest link in the power flow chain.[1,2] Insulators in such generators are stressed to their limits to take advantage of the power densities that are achievable in the other components of the system. Hence, it is desirable to know how much voltage can an insulator reliably withstand. This is especially important in large and complex multielement systems where a high overall reliability is required.

Accelerator designers have been very well served by an empirical formula[3] for short pulse vacuum insulator breakdown ($t < 1$ μs) developed by J. "Charlie" Martin at Atomic Weapons Research Establishment at Aldermaston, UK. This formula yields the mean breakdown field for an insulator as a function of the insulator area and the duration of the electrical pulse applied to the insulator.

An important message from Martin's original note[3] is that the inherent scatter in the breakdown data (not arising from measurement uncertainties) implies that the breakdown strength of the insulator decreases with increasing area. Martin also suggested that while the normalized standard deviation for the breakdown strength is independent of the insulator area, the functional dependence of the breakdown strength on area could be extracted from the standard deviation in the breakdown data.

Treating insulator breakdown as the failure of a multilink chain where the failure of one link causes the failure of the whole chain couples these apparently disconnected facts and forms the basis of a statistical analysis associated with the name of Weibull.[4,5]

In this paper, we discuss the applicability of Weibull statistics to analyze insulator breakdown data and use as an example data obtained on a 75 cm^2 insulator sample. It is quite possible that other data already exist elsewhere. It would be of great benefit to the pulse power community if these data could be consistently analyzed. The data could reveal the insulator design improvements likely to produce a better performance.

The J. C. Martin Breakdown Relationship

The J. C. Martin insulator breakdown (JCM) relationship and its variants have been applied in the pulsed power field for the past decade. Successful designs for vacuum envelopes of many accelerators with a wide range of voltages and pulse lengths attest to its applicability.

However, nagging questions always remained after its use by the insulator designer, i.e.: how reliable would the insulator be in terms of the percentage of insulator flashovers that could be expected at a given voltage; and did insulator failures occur in practice at a frequency that was expected or did they occur more frequently warranting a revision of the design or operating environment of the insulator.

Two generic types of insulators are in use today.[1,6] One type are stacked insulators (Fig. 1(a)), usually incorporated in systems that operate above 1.5 MV. They consist of a stack of alternating plastic cone frustra and metal gradient rings. The inner faces of the rings are angled from 45 to 60 degrees such that electrons emitted by metal surfaces do not impact the insulator.

The JCM breakdown relationship[3] for the 50% failure probability of a stacked Lucite 45° insulator is:

$$E\, t^{1/6} A^{1/10} = 175$$

where E is the flashover field in kilovolts/centimeter, A is the area of the insulator in square centimeters, and t is the time in microseconds the insulator is stressed to greater than 89% of the maximum field value. Comparative strengths for several materials as a function of angle[8,9] appear in the literature.

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14. ABSTRACT
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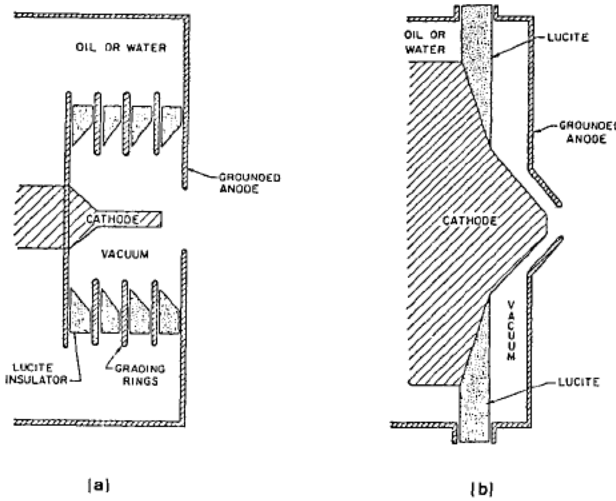


Figure 1. High voltage insulators for pulsed applications.[6] (a) Comprises a stack of insulator cone frustra and grading rings, (b) is a diaphragm type insulator.

The JCM relationship assumes that standard practices in the design of a stacked insulator have been followed, i.e.: providing a recess in the anode side of the gradient ring to reduce the electric field at the anode triple point (vacuum-insulator-gradient ring) junction; extending the gradient ring beyond the insulator edge to provide some electron and vacuum UV shielding; guaranteeing an intimate, void free contact between the base of the insulators frustra and the gradient rings; and distributing the potentials uniformly over the stack with a 10% uniformity goal. Ref. 10 discusses these practices in more detail.

The other type of vacuum envelope used in lower voltage accelerators appears in Fig. 1(b). This is called a diaphragm vacuum envelope. The surfaces are arranged such as to have the electric field be within 20° to the normal to the surface and uniform across the radius except in the vicinity of the anode and cathode. This paper endeavors to establish the breakdown relationship for a uniform field insulator assuming a 1/6 power time dependence.

Weibull Statistics

The decrease in the insulator strength associated with an increase in area suggests a model for insulator failure that considers the insulator as a chain where the probability of survival of the whole chain is the product of the probability of survival of the individual links. Consequently, we can view an insulator of a given area as the parallel or series connection of many samples of smaller areas such that the survival probability of the whole insulator is the product of the survival probabilities of the samples.

Let us suppose that $R(S/\bar{S})$ be the probability that a sample with a mean breakdown stress $\bar{S}$ will survive an applied stress S. Suppose we now take N such samples in series or in parallel. The probability that the N samples will survive is:

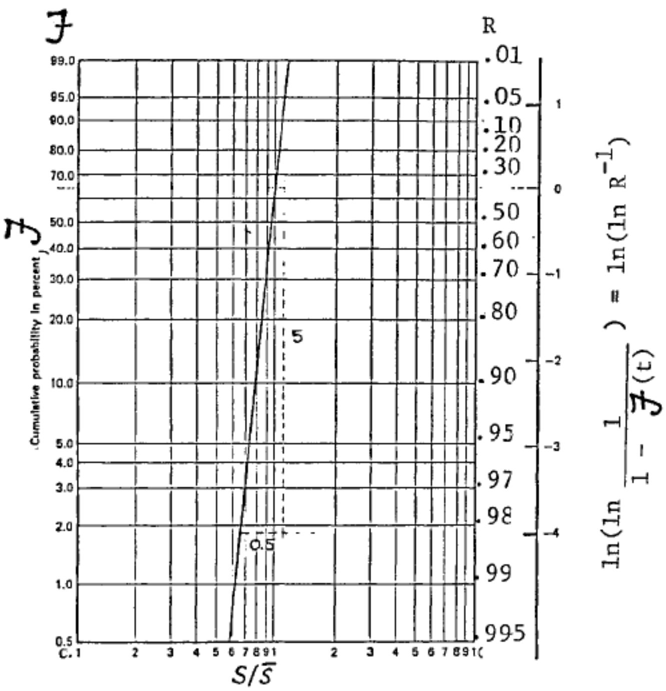
$$R(S/\bar{S})^N = R(S/\bar{S}_N)$$

From the JCM relationship, let the area exponent be 1/B (B = 8-10). Then, $\bar{S}_N = \bar{S}N^{-1/B}$. Consequently, $[R(S/\bar{S})]^N = R(N^{1/B}S/\bar{S})$. A function that has this property is:

$$R(S/\bar{S}) = e^{-(S/\bar{S})^B}$$

The failure probability is then $\mathcal{F} = 1 - R(S/\bar{S})$. $\mathcal{F}$ is called the Weibull failure distribution function[5] or cumulative failure distribution. It was also derived independently by Creedon.[11]

A plot of $\mathcal{F}$ as a function of $S/\bar{S}$ for B = 10 appears in Fig. 2. The grid dimensions are set up such that for B = 1, $\mathcal{F}$ is a straight line with a 45° slope. In the Weibull plot, the probability of failure in the interval $0 \le S/\bar{S} \le 1$ is equal to $F = 1 - e^{-1} = 0.632$.



APPLIED STRESS

Figure 2. Cumulative failure distribution or Weibull failure distribution function for B = 10. B is the geometric slope. (From Fig. 11.2 of Ref. 5.)

From the Weibull cumulative failure distribution

$$\mathcal{F} = 1 - e^{-(S/\bar{S})^B}$$

it is possible to obtain the probability distribution function (pdf) by differentiation:[5]

$$f = \frac{B}{\bar{S}} \left(\frac{S}{\bar{S}}\right)^{B-1} e^{-(S/\bar{S})^B}$$

where f(S)dS is the probability that breakdown will occur in the interval (S,S + dS). The mean (first moment of the pdf) becomes:

$$\frac{\mu}{\bar{S}} = \Gamma\left(1 + \frac{1}{B}\right)$$

and the standard deviation (moment of the square of the difference between the pdf and the mean becomes

$$\left(\frac{\sigma}{\bar{S}}\right)^2 = \left[\Gamma\left(1+\frac{2}{B}\right) - \Gamma^2\left(1+\frac{1}{B}\right)\right]$$

where Γ is the usual gamma function.

The value of $\mu/\bar{S}$ and $(\sigma/\bar{S})^2$ appear in the table below:[5]

B	$\mu/\bar{S}$	$(\sigma/\bar{S})^2$
0.5	2.0000	20.000
1.0	1.0000	1.000
2.0	0.8862	0.215
3.0	0.8934	0.105
3.5	0.8998	0.081
4.0	0.9064	0.065
5.0	0.9182	0.044
6.0	0.9275	0.033
10.0	0.9514	0.013
20.0	0.9730	0.004

The expressions for the mean and standard deviation complement two empirically known facts:

a) The mean normalized breakdown field $\mu/\bar{S}$ is independent of the area and depends on the exponent B only.

b) That the variance $\left(\frac{\sigma}{S}\right)^2$ depends only on the value of the exponent B in the JCM expression.

In the case of insulator breakdown, we take $S = Et^{1/6}$ where E is the applied field and t is the time the field exceeds 89% of its maximum value.

The time dependence of breakdown for insulators where the area is held constant may be treated in a similar manner. For the time dependence, we may consider an insulator of fixed area A subject to a stress $EA^{1/10}$ with a duration t. A stress duration Nt is then equivalent to stressing N insulators for a time t. The exponent B for the time dependence in the JCM formulation is equal to 6.

Martin's analysis of insulator breakdown data is consistent with the Weibull statistical analysis. With 1/10 as the area exponent, (B = 10) Table 1 shows that $\sigma_A/\bar{S}_A \approx 11\%$ $(\bar{S}_A = \overline{Et^{1/6}})$ and with 1/6 as the time exponent, $\sigma_t/\bar{S}_t \approx 18\%$ $(\bar{S}_t = \overline{EA^{1/10}})$. These numbers are quite close to the 10% and 15% quoted in Martin's[3] note.

Experimental Apparatus

The vacuum region of the DNA POCO facility at Maxwell Laboratories used to gather the breakdown data appears schematically in Fig. 3(a). A 10 Ω, 30 ns water dielectric pulse forming line drives a 10 Ω, 30 ns water output line providing a 60 ns wide output pulse with a 20 ns risetime as shown by the solid waveform of Fig. 3(b). The dashed waveform corresponds to an insulator breakdown. A $\dot{D}$ probe located in the water immediately before the main insulator measured the voltage.

Table 1. Insulator Breakdown Data. $\langle Et^{1/6}\rangle$ originates from Fig. 5. The fraction failing $\mathcal{F}$ corresponds roughly to the data point number expressed fractionally, i.e., 13/26 = 0.5 ≈ 0.48. The mean breakdown strength $\bar{S}$ = 133 yields $\ln(\langle Et\rangle^{1/6}/133) = 0$ when $\mathcal{F} = 0.632$.

Data Point Number	$\langle Et^{1/6}\rangle$ (kV $\mu s^{1/6}$)	Fraction Failing $\mathcal{F}$	$\ln\left(\frac{\langle Et^{1/6}\rangle}{133}\right)$	$\ln\left(\ln\frac{1}{1-}\right)$
1.000	92.839	0.027	-0.359	-3.617
2.000	99.852	0.064	-0.286	-2.710
3.000	101.937	0.102	-0.266	-2.227
4.000	102.667	0.140	-0.259	-1.890
5.000	106.114	0.178	-0.225	-1.629
6.000	106.882	0.216	-0.218	-1.414
7.000	113.704	0.254	-0.156	-1.228
8.000	113.972	0.292	-0.154	-1.065
9.000	114.642	0.330	-0.148	-0.917
10.000	118.306	0.367	-0.117	-0.781
11.000	118.984	0.405	-0.111	-0.654
12.000	119.292	0.443	-0.108	-0.535
13.000	119.833	0.481	-0.104	-0.422
14.000	121.490	0.519	-0.090	-0.312
15.000	121.508	0.557	-0.090	-0.206
16.000	123.309	0.595	-0.075	-0.102
17.000	129.937	0.633	-0.023	0.001
18.000	134.070	0.670	0.008	0.104
19.000	134.676	0.708	0.013	0.209
20.000	138.925	0.746	0.044	0.316
21.000	140.093	0.784	0.052	0.427
22.000	140.744	0.822	0.057	0.546
23.000	146.430	0.860	0.097	0.676
24.000	147.731	0.898	0.105	0.824
25.000	152.965	0.936	0.140	1.009
26.000	165.310	0.973	0.218	1.289

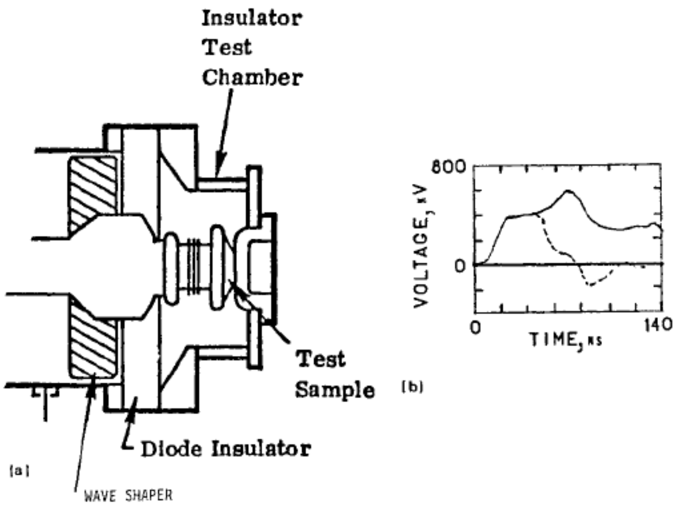


Figure 3. (a) Vacuum region of modified POCO Facility. Insulator replaces conventional field emission diode. Detail of insulator under test apears in Fig. 4. (b) Voltage applied to test insulator. Dashed waveform corresponds to a breakdown.

Details of the insulator under test appear in Fig. 4. Field shaping electrodes provide a uniform electric field along the insulator surface as encountered on diaphragm type insulators.

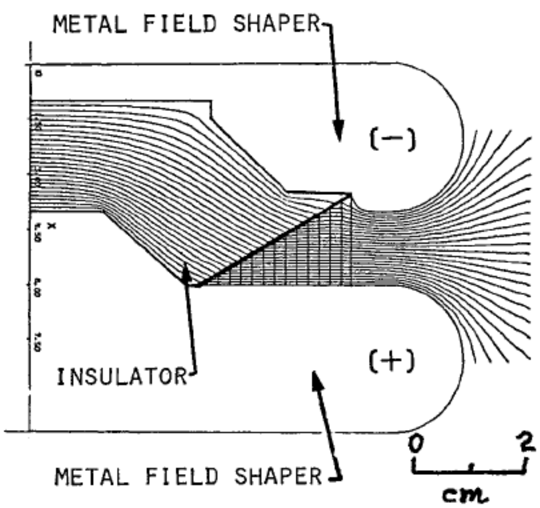


Figure 4. Detail of insulator geometry. Electrode profiles provide a uniform field along the face of face insulator. Insulator material is polymethylmetracrylate (PMMA or Plexiglas).

Experimental Results and Data Reduction

Data from 26 experiments ($Et^{1/6}$) displayed as the solid circles in Fig. 5 suggest an improving trend, in the average, of insulator strength with successive voltage applications. To remove the effects of this conditioning the data, the function $\bar{E}t^{1/6} = A_0 + A_1 m + A_2 m^2$ was fit to the data. The sequential number of the experiment was m and $A_0 = 124$, $A_1 = 2.45$, and $A_2 = 0.05$ are obtained by a least squares fit. The open circle data was then reduced as $\langle Et^{1/6}\rangle = \frac{A_0\ Et^{1/6}}{\bar{E}t^{1/6}}$. Table 1 ranks the data in ascending order of values of $\langle Et^{1/6}\rangle$. The fraction failing $\mathcal{F} = \frac{n - 0.3}{N + 0.4}$ represents a median rank accounting for the limited size of the sample.[5] Note that as $N \to \infty$, $\mathcal{F} = n/N$.

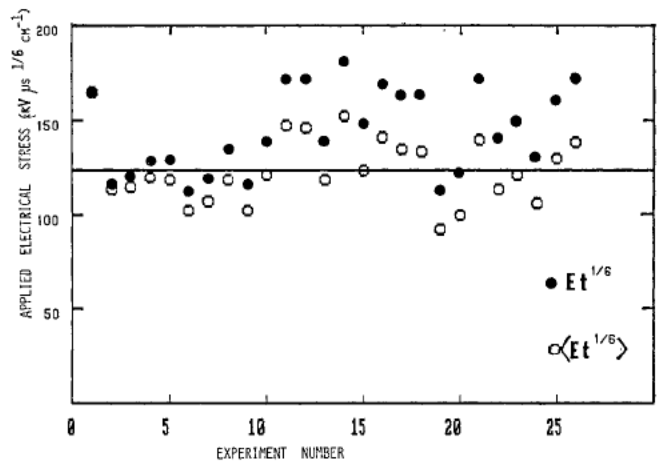


Figure 5. Experimental breakdown data (solid circles), and normalized breakdown data (open circles) as a function of consecutive shot numbers. Normalization has eliminated conditioning effect on insulator.

The other entries of the table are used to construct the Weibull plot of Fig. 6. The mean breakdown stress of the insulator $(\overline{\langle Et^{1/6}\rangle} = 133)$ corresponds to $\mathcal{F} = 0.63$. With the conventional interpretation, (50% breakdown probability, $\mathcal{F} = 0.5$), the JCM breakdown formula for such a uniform field insulator becomes $Et^{1/6}\ A^{1/8} = 218$ ($\ln Et^{1/6}/\bar{S} = -0.45$, $\bar{S} = 133$, and $A = 75\ cm^2$).

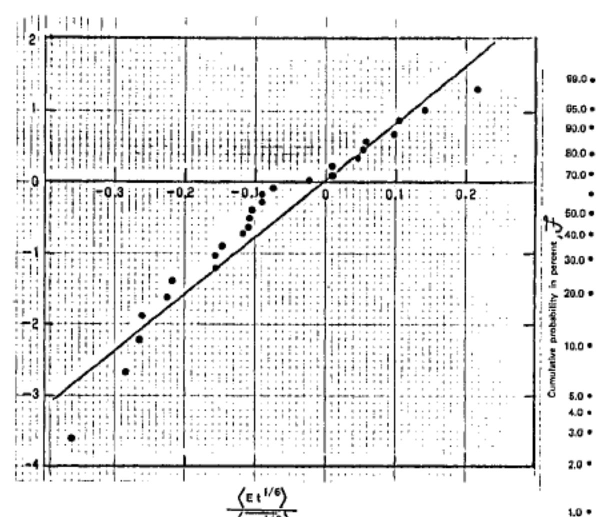


Figure 6. Weibull plot of insulator data. Dependence on stress is 8th power. The ordinate is zero for = 0.632 corresponding to $\bar{S} = 133$.

Design Example

As a design example, suppose we want to construct, based on the 75 cm^2 data, a 10^4 cm^2 insulator that would breakdown, over a long experimental run, 10% of the pulses for a 60 ns pulse.

For $\mathcal{F} = 0.1$, $\ell n\, \ell n\, (1/0.9) = -2.25$, $\ell n(Et^{1/6}/\bar{S}) = -0.284$, and $Et^{1/6}/\bar{S} = 0.75$. Using $\bar{S} = 133$ for a 75 cm^2 insulator, we obtain $\bar{S} = 133(75/10^4)^{1/8} = 72$ for a 10^4 cm^2 insulator. Consequently, $Et^{1/6} = 54.1$. For t = 0.06 μs, $E = 86$ kV cm^{-1}.

Conclusions

Breakdown data for a uniform field insulator, analyzed using Weibull statistics, yields a power dependence on the field equivalent to a 1/8 power dependence in the area. The data may be used to scale uniform field insulators. Weibull statistics may also be used to estimate the reliability of stacked insulators designed with the JCM breakdown formula.

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